$x_1 + 2x_2 = v_1$

$-3x_2 = -2v_1 + v_2$

$x_2 = \frac{2v}{3} - \frac{v_2}{3}$

$\rightarrow x_1 + 2\left(\frac{2v_1}{3} - \frac{v_2}{3}\right) = v_1$

$x_1 + \frac{4v_1}{3} - \frac{2v_2}{3} = v_1$

$3x_1 + 4v_1 - 2v_2 = v_1$

$3x_1 - 2v_2 = -3v_1$

10/7 Lecture Notes

(do row reduction practice problems)

Span = Let $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_m$ be vectors in $\mathbb{R}^n$. The span of these vectors is the set of all linear combinations of those vectors (lin. comb $= c_1u_1, \ldots c_mu_m$ where c is any constant you like)

Ex] Is $\begin{bmatrix}2\\5\\3\end{bmatrix}$ in span$\left(\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}0\\1\\1\end{bmatrix}\right)$? What about $\begin{bmatrix}1\\1\\1\end{bmatrix}$

The vector $\begin{bmatrix}1\\1\\0\end{bmatrix}$ can be thought of in 3-space as the $y=x$ line & $\begin{bmatrix}0\\1\\1\end{bmatrix}$ as $y=z$ line. "Is this in this span?" Means is there a linear combination of $\begin{bmatrix}1\\1\\0\end{bmatrix}$ & $\begin{bmatrix}0\\1\\1\end{bmatrix}$ that equals $\begin{bmatrix}2\\5\\3\end{bmatrix}$ $\left\{ \begin{bmatrix}1 & 0 & | & 2\\1 & 1 & | & 5\\0 & 1 & | & 3\end{bmatrix} \sim \begin{bmatrix}1 & 0 & 2\\0 & 1 & 3\\0 & 0 & 0\end{bmatrix} \right.$ therefore: $(2)\begin{bmatrix}1\\1\\0\end{bmatrix} + (3)\begin{bmatrix}0\\1\\1\end{bmatrix} = \begin{bmatrix}2\\5\\3\end{bmatrix}$ so yes, it is in the span.

For $\begin{bmatrix}1\\1\\1\end{bmatrix}$ the answer is No because: $\begin{bmatrix}1 & 0 & | & 1\\1 & 1 & | & 1\\0 & 1 & | & 1\end{bmatrix} \sim \begin{bmatrix}1 & 0 & | & 1\\0 & 1 & | & 1\\0 & 1 & | & 0\end{bmatrix}$ } This has No solutions.

(For $\begin{bmatrix}0\\1\\1\end{bmatrix}$ & $\begin{bmatrix}1\\1\\0\end{bmatrix}$ specifically, the middle term will always have to be a sum of the 1st & last, eg $\begin{bmatrix}2\\5\\3\end{bmatrix}$ but not $\begin{bmatrix}1\\1\\1\end{bmatrix}$

The Span is Not $\mathbb{R}^2$!!!! It is a plane. But thats not the same thing...

Theorem: Let $\vec{u}_1, \vec{u}_2, \ldots, \vec{u}_m, \vec{v}$ be vectors in $\mathbb{R}^n$. $\vec{v}$ is in span$(\vec{u}_1, \ldots, \vec{u}_m)$ if and only if there is a solution to the linear system corresponding to the augmented matrix $[\vec{u}_1 \ldots \vec{u}_m \mid \vec{v}]$

Span of two vectors $\vec{v}$ & $\vec{w}$ is the set of all their linear combinations $a\vec{v} + b\vec{w}$, so for vectors on the xy plane thats just all of the 2d xy plane (unless they line up, in which case their span is just the line they lie on)

So, 2 vectors in $\mathbb{R}^3$ make a plane, 3 vectors in $\mathbb{R}^3$ creates a span of every possible vector in $\mathbb{R}^3$ (unless of course 1 of those vectors lies along the same span as the first two)

If one of the vectors lines up with the span of another(s), then it doesn't add anything to the span, and is therefore linearly dependent.

Linearly independent - vectors that do add something to the span

If we have a solution $x_1 = c_1, \ldots, x_m c_m$ to $[\vec{u}_1, \ldots, \vec{u}_m \mid \vec{v}]$, then $c_1\vec{u}_1 + \ldots c_m\vec{u}_m = \vec{v}$, so $\vec{v}$ is in the span$(u_1, \ldots, u_m)$

Ex] Find a vector $\vec{v}$ that is not in the span of $\left(\begin{bmatrix}1\\1\\0\end{bmatrix}, \begin{bmatrix}-1\\1\\2\end{bmatrix}, \begin{bmatrix}5\\1\\-4\end{bmatrix}\right)$ that is in $\mathbb{R}^3$, or show span is all of $\mathbb{R}^3$.

Looking for solutions to the system corresponding to aug. matrix: $\begin{bmatrix}1 & -1 & 5 & | & v_1\\1 & 1 & 1 & | & v_2\\0 & 2 & -4 & | & v_3\end{bmatrix}$

$R_2 = -R_1 + R_2$ $\begin{bmatrix}1 & -1 & 5 & | & v_1\\0 & 2 & -4 & | & -v_1+v_2\\0 & 2 & -4 & | & v_3\end{bmatrix} \sim$ $R_3 = -R_2 + R_3$ $\begin{bmatrix}1 & -1 & 5 & | & v_1\\0 & 2 & -4 & | & -v_1+v_2\\0 & 0 & 0 & | & v_1-v_2+v_3\end{bmatrix} \Rightarrow$ This means there are NO solutions when $v_1 - v_2 + v_3 \neq 0$, eg $\begin{bmatrix}0\\0\\4\end{bmatrix}$ is not in span